

A Redshift Dependent Color-Luminosity Relation in Type 1a Supernovae

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ABSTRACT

We find a highly significant correlation of Type 1a supernova magnitudes in the Union 2.1 compilation of 580 sources. The correlation of magnitude residuals relative to the Λ CDM model and $color \times redshift$ has a significance equivalent to 13 standard deviations, as evaluated by randomly shuffling the data. We generalize the standard $B - V$ color correction to include a Taylor series in redshift z . The goodness of fit χ^2 decreases by more than 50 units using one additional parameter linear in $color \times redshift$. The new parameter shifts the supernova best-fit cosmological dark energy density parameter from $\Omega_\Lambda = 0.71 \pm 0.02$ to $\Omega_\Lambda = 0.74 \pm 0.02$ assuming a flat universe. Varying Ω_m and Ω_Λ separately produces $\Omega_m + \Omega_\Lambda = 1$ within errors. The $color - redshift$ correlation is quite robust, cannot be attributed to outliers, and passes several tests indicating it does not originate in data selection or systematic error assignments. One physical interpretation is that supernovae or their environments evolve significantly with increasing redshift. The previously known rule that bluer supernovae have larger absolute luminosity tends to flatten out observationally with increasing redshift.

Subject headings: Supernovae: general; cosmology: dark matter;cosmology:dark energy; ; cosmology:cosmological parameters

1. Introduction

Observations of Type 1a supernovae provide evidence for an accelerating expansion of the universe and dark energy. Supernovae are imperfect standard candles, and the corrections for their intrinsic luminosity has evolved over time. Phillips (Phillips 1993) early observed the important law relating absolute magnitudes and time scales or “stretch factors.” In 1998 Tripp discovered a color correction parameter which greatly improved the model fit for the 29 distant Type Ia supernovae available at the time(Tripp 1998) . The sense of Tripp’s correction is that type 1a supernovae with bluer colors tend to be intrinsically brighter. Currently several parameters of magnitude, stretch, color, galactic environment, etc. are fit alongside the cosmological parameters of dark matter and dark energy density. Global fits couple all the parameters, so that the parameters correcting absolute magnitudes directly affect the dark matter and dark energy density.

In the Union 2.1 compilation (Suzuki et al. 2012) the distance modulus μ_B corrected for color (c), stretch (x_1) and a certain probability $P_m = P(m_*^{host} < m_*^{Threshold})$ of the host galaxy is

$$\mu_B(\alpha, \beta, \delta, M_B) = m_B + \alpha x_1 - \beta c + \delta P_m - M_B. \quad (1)$$

Here $x_1 = s - \bar{s}$, where s is the time stretch factor after redshift (z) corrections have been applied (Goldhaber et al. 2001; Guy et al. 2007). Symbol $c = color - \overline{color}$, where $color = (B - V)_{max} + 0.057$ and the Johnson-Cousins B stands for blue and V the visual magnitude. Overbars denote mean values. We have also subtracted the mean from P_m to remove a degeneracy affecting the value of M_B .

To test whether supernovae magnitude relations depend on redshift, we generalize the existing parameters to a Taylor series expansion in powers of $1 + z$. We then re-fit the data to the standard Λ CDM Friedman-Lemaître-Robertson-Walker (FLRW) framework. The *color* correction parameter β received our main attention due to its astrophysical interpretation. Our generalization is

$$\beta c \rightarrow \beta(z)c = \beta_0 c + \beta_1 (cz - \overline{cz}). \quad (2)$$

Our study reveals a correlation of *color* parameters with *redshift* of very high statistical significance. The residuals of Union 2.1 data relative to Λ CDM cosmology are correlated with $c \times z$ at the level equivalent to a 13σ Gaussian fluctuation. Introducing a single parameter β_1 and fitting the magnitudes of the 580 *Sn1a* to a Λ CDM cosmology reduces the χ^2 value by more than 50 units compared to previous fits assuming $\beta_1 = 0$. For reference we call this empirical correlation the “*color – redshift* effect.”

1.1. Definitions

Conventional analysis assumes a Λ CDM cosmology with zero radiation density, dark energy density Ω_Λ and dark matter density Ω_m . The model’s predicted luminosity distance is (Weinberg 2008. p38-55)

$$d_L(z) = \frac{c(1+z)}{H_0\sqrt{\Omega_k}} \sinh \left(\sqrt{\Omega_k} \int_{1+z}^1 \frac{dx}{x^2 H(x)} \right);$$

$$\text{where } H(x) = \sqrt{\Omega_\Lambda x^{-3(1+w)} + \Omega_m x^{-3}}, \quad (3)$$

with w regulating the dark energy equation of state and H_0 the Hubble constant (here $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$). The flat cosmology has $\Omega_k = 1 - \Omega_m - \Omega_\Lambda = 0$. We restrict study here to the standard value $w = -1$. Our preliminary studies find no great sensitivity between the new effects and w .

The distance modulus μ_{model} is defined by

$$\mu_{model}(z, \Omega_\Lambda, \Omega_m) = 5 \log_{10} \left(\frac{d_L(z)}{\text{Mpc}} \right) + 25. \quad (4)$$

1.2. Data and Analysis

We actually began with the 2008 Union compilation of Kowalski *et al.* (Kowalski et al. 2008). By interpreting the cuts we were able to obtain the 307 *SN1a* reported as the 3σ set. Yet the final data tables of this compilation gave only raw magnitude uncertainties, in which extensive corrections for systematic errors were not included. We found a large *color* – *redshift* correlation, and explored a fit using the raw magnitude uncertainties. One *color* \times *redshift* parameter β_1 yielded more than 30 units of χ^2 improvement. We then studied the Union 2.0 compilation of Amanullah *et al.* (Amanullah et al. 2010), which gave both raw and total uncertainties, while it does not use the δ parameter of Union 2.1. This paper, like (Kowalski et al. 2008), lacked sufficient detail for all of its results to be reproducible by us. Nevertheless the improvement of χ^2 per data point using β_1 and the reported uncertainties was found to be quite comparable. Finally the Union 2.1 compilation of (Suzuki et al. 2012) is one of the largest and most recent collections, which subsumes many of the earlier *SN1a* compilations. The publication gave enough detail to reproduce its χ^2/df using its best-fit parameters $\alpha, \beta, \delta, \Omega_m, M_B$ and $w = -1$. Our results here are confined to the “ 3σ set” of Union 2.1 data passing certain quality cuts described below.

In all of our studies the processes of data reversion and analysis calculations were done twice, by independently written programs that compared outputs while sharing no common elements of code.

2. Correlation Results

We first fit redshift-independent parameters $\alpha_0, \beta_0, \delta_0, M_B$, and Ω_m to represent the standard Λ CDM model with equation of state parameter $w = -1$ and $\Omega_\Lambda = 1 - \Omega_m$. Results are shown in the top line of Table 1.

We will first discuss residuals δ_μ , which are given by the differences between the distance moduli and the model:

$$\delta_\mu = m_B - \mu_{model}. \quad (5)$$

The residuals of the fit versus *color* \times *redshift* are shown as a scatter plot in Figure 1. The correlation is readily seen by eye.

The Pearson coefficient r quantifies the correlation. It is defined by

$$r(x, y) = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}} \quad (6)$$

We find $r_{SN} = r(\delta_\mu, c z) = -0.52$. The significance of r_{SN} was estimated by comparing a Monte-Carlo simulation using the data itself. The simulation randomly shuffles the (*color* \times *redshift*) data elements, re-calculates a value r_{random} , and saves it. Figure 2 shows the histogram of r_{random} from

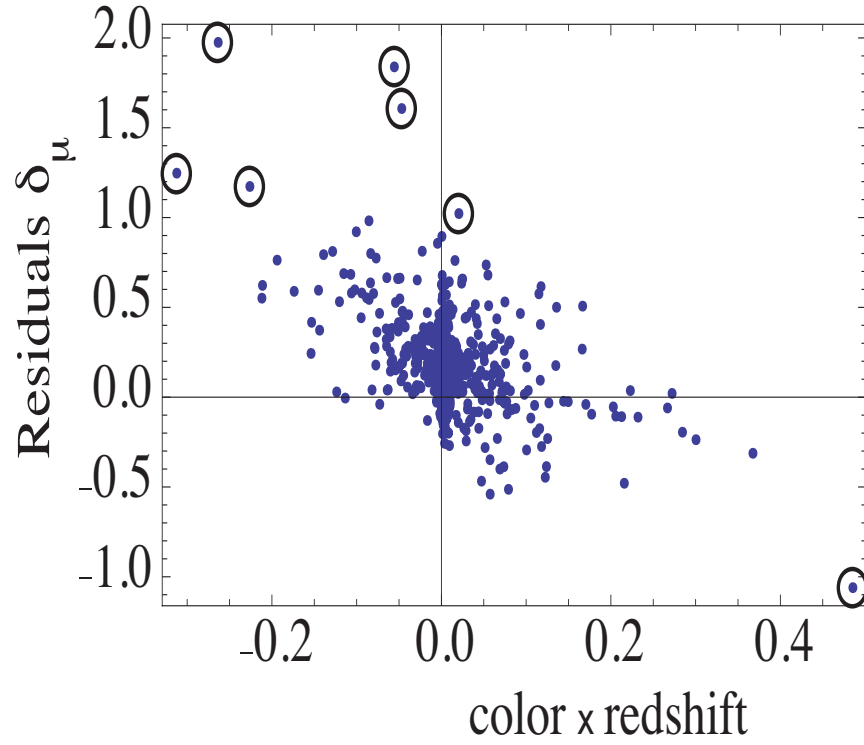


Fig. 1.— Distance modulus residuals $\delta_\mu = m_B - \mu_{model}$ versus *color* \times *redshift* from the best-fit Λ CDM model of the Union 2.1 data compilation with a flat universe and $w = -1$. The Pearson correlation of $r = -0.52$ has the significance of a 13σ effect, as evaluated by a simulation shuffling the data randomly. Residuals are defined by $\delta_\mu = m_B - \mu_{model}$. Seven points discussed in the text are indicated by circles.

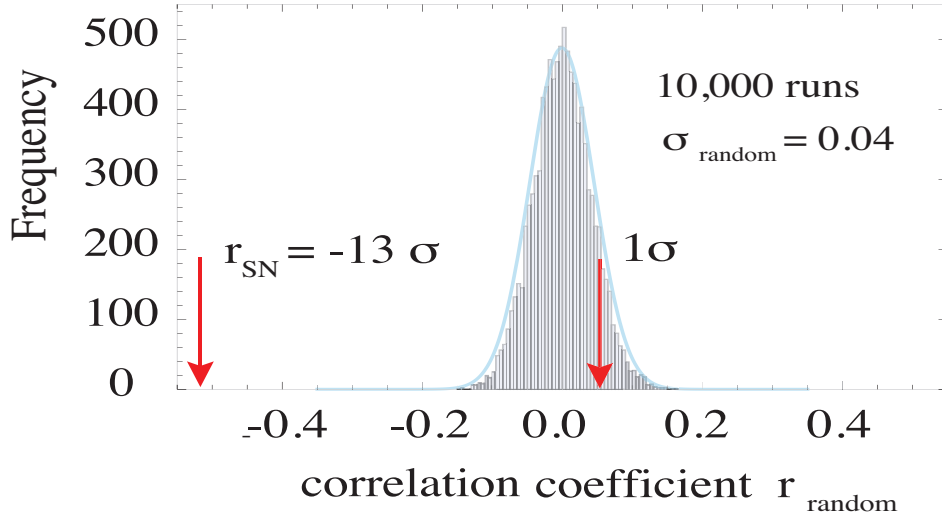


Fig. 2.— Histogram of random Pearson correlations r_{random} obtained by shuffling $(\text{color} \times \text{redshift})$ parameters of the data relative to its residuals. The value r_{SN} computed with the data is about 13σ from the mean. The curve shows a Gaussian distribution with mean $\bar{r}_{\text{random}} = -0.00067$ and standard deviation $\sigma_{r-\text{random}} = 0.042$.

10,000 runs. The mean and standard deviation of the random correlations are $\bar{r}_{\text{random}} = -0.00067$ and $\sigma_{r-\text{random}} = 0.042$. The data's correlation of r_{SN} is about $13\sigma_{r-\text{random}}$ from the mean of the random correlations. The estimated P -value (of order 10^{-39}) is too small to simulate or interpret as a fluctuation.

One might ask whether the correlation is dominated by a few outliers. Actually the Union 2.1 set we are using has already rejected outliers. Starting with 753 *Sn1a* in the full compilation, points further than 3σ from the model prediction were discarded, leaving the 580 points we use. The cuts also include (1) that the CMB-centric redshift is greater than 0.015; (2) there must be at least one point between -15 and 6 rest-frame days from B-band maximum light; (3) At least five valid data points must exist; (4) the entire 68% confidence interval for x_1 must lie between -5 and +5; (5) data must come from at least two bands with rest-frame central wavelength coverage between 2900 \AA and 7000 \AA ; (6) at least one band must be redder than rest-frame U-band (4000 \AA). As an experiment we explored removing the seven points with largest $|\delta_\mu|$ and indicated with circles in Figure 1. That reduced the correlation to $r_{\text{SN}, \text{cut}} = -0.45$, an 11σ effect.

2.1. A Model with One New Parameter:

We define our fit statistic χ^2 as

$$\chi^2 = \sum_{SNe} \frac{(\mu_B - \mu_{model})^2}{\sigma_\mu^2} \quad (7)$$

with σ_μ being the complete (statistical and systematic) errors provided in the full table on the SCP website¹. With standard, redshift-independent color correction the best fit value is $\chi^2 = 550$ for $580 - 5$ degrees of freedom (df) (Table 1). Since nothing in our study depends on a fraction of a unit of χ^2 we round the values to the nearest integer. Allowing for redshift-dependent color correction, $\beta c \rightarrow \beta_0 c + \beta_1 (cz - \overline{cz})$ finds a best fit with $\chi^2 = 500$ with $(580 - 6)df$. That is a 50 unit improvement in the fit from the addition of one parameter β_1 : see Table 1. Note the $\beta_1 \neq 0$ model is smoothly connected to the formal “null hypothesis” that no color-redshift correlation exists. Then Wilks’ Theorem predicts $\Delta\chi^2$ is distributed by χ_1^2 in the null, for which a 50 unit fluctuation has a chance probability of order 10^{-12} .

Letting Ω_M and Ω_Λ vary independently we find $\Omega_m + \Omega_\Lambda = 1$ within errors. Compared to assuming a flat universe *a priori* the goodness of fit is improved less than a unit of χ^2 by letting Ω_Λ be a free parameter. Alternatively, one cannot improve the fit by adding $\Omega_k = 1 - \Omega_M - \Omega_\Lambda$ as a parameter.

Achieving such a low χ^2/df would be unusual on a statistical basis. However there is a simple explanation. In developing systematic errors there is a step adjusting the reduced χ^2/df of each sample to unity. (See (Suzuki et al. 2012) following Eq. 7). We used the errors published and made no sample-by-sample corrections, but doing so after β_1 is fit would trivially bring χ^2/df back to one. There is a question of whether the method of assigning systematic errors, which is iterative and tuned to the model, might have a role in the *color – redshift* effect. That is impossible for us to resolve. However Section 3 discusses a simple unbiased procedure that gives evidence the assignment of systematic errors should not be a decisive feature of our findings. In passing we note that the constant M_B is not independently observed in simple *Sn1a* fits, where its effects are degenerate with the Hubble parameter. Related to this, the traditional absolute magnitude M_B has been called a “nuisance parameter”, and many papers have come to omitting the value obtained from their fits. We do not agree that M_B or the other parameters are devoid of physical meaning, and Table 1 reports all parameters needed to reproduce our results.

Figures 3 and 4 show χ^2 versus Ω_m and Ω_Λ with and without accounting for the *color* \times *redshift* effect. The plots are made with parameters $\alpha_0, \beta_0, \delta_0, M_B$ evaluated at their best-fit values point by point and $\Omega_m + \Omega_\Lambda = 1$. The important cosmological parameters Ω_m and Ω_Λ are sensitive to value of β_1 . The significance of the *color – redshift* effect for Ω_m and Ω_Λ depends on how it is computed and assessed. For example, a similar plot with the other parameters fixed

¹The full Union 2.1 data files are available at <http://supernova.lbl.gov>

Ω_m	Ω_Λ	α_0	β_0	δ_0	M_B	β_1	χ^2_{min}	$\Delta\chi^2$
0.291 ± 0.022	$1-\Omega_m$	0.105 ± 0.007	2.31 ± 0.05	-0.022 ± 0.03	-19.133 ± 0.013	0^*	550	0
0^*	0^*	0.10 ± 0.006	2.51 ± 0.07	-0.11 ± 0.024	-19.1 ± 0.012	-1.35 ± 0.22	534	16
0^*	0.31 ± 0.003	0.105 ± 0.00004	2.61 ± 0.005	-0.054 ± 0.0007	-19.05 ± 0.0002	-1.59 ± 0.051	511	39
0.260 ± 0.021	$1-\Omega_m$	0.102 ± 0.007	2.62 ± 0.07	-0.038 ± 0.03	-19.14 ± 0.013	-1.61 ± 0.23	500	50
0.259 ± 0.07	0.737 ± 0.13	0.102 ± 0.0065	2.62 ± 0.07	-0.038 ± 0.027	-19.16 ± 0.073	-1.61 ± 0.227	500	50

Table 1: Comparison of fits without accounting for the *color – redshift* effect ($\beta_1 = 0$) and including it ($\beta_1 \neq 0$). Parameters held fixed are indicated by an asterisk. The *color – redshift* effect produces a highly significant improvement of the fit even with the unphysical constraint $\Omega_m = \Omega_\Lambda = 0$. χ^2_{min} and $\Delta\chi^2$ have been rounded to the nearest whole number.

to the global best fit value (Mohlabeng & Ralston 2012) finds that Ω_m and Ω_Λ shift by more than their 99.95% (3σ) confidence level uncertainties. That is appropriate when other parameters are known. The plots here showing χ^2 with other parameters floating to their best-fit values are more conservative, and based on the supernova data alone. (For example, no further information from CMB or galaxy distributions has been assumed. It would be interesting to explore the effects of joint fits.) The errors in the Table 1 come from the 6×6 or 7×7 covariance matrices using the convention² of $\Delta\chi^2 = 1$.

2.1.1. Other Redshift-Dependent Parameters:

Random searches will dilute statistics, and we did not make many before finding the *color – redshift* effect. After we found its significance was high we considered whether other parameters might be a cause or contributing factor. The fits involve integrations and are computationally slow. Thus we found it more orderly (if not exhaustive) to fix parameters and look for redshift dependence of other parameters sector by sector. The variations and their results are:

$$\begin{aligned}
 (\text{fix } \beta_1 = \delta_1 = 0) : \alpha &\rightarrow \alpha_0 + \alpha_1 z; \quad \alpha_1 = -0.036; \Delta\chi^2 = 3.8; \\
 (\text{fix } \alpha_1 = \beta_1 = 0) : \delta &\rightarrow \delta_0 + \delta_1 z; \quad \delta_1 = -0.093; \Delta\chi^2 = 2.
 \end{aligned} \tag{8}$$

The *a-posteriori* justification for varying parameters one at a time is that none but β_1 produce very significant effects.

We considered whether a z -dependent stretch correction α_1 might be redundant. The details of extracting x_1 from case-by-case fits are not available to us, which might create a danger of double-counting. Similarly the systematic errors used in χ^2 fit were determined by complicated

²Multi-parameter confidence levels can be assessed using several conventions. The $\Delta\chi^2 = 1$ convention follows the text of Ref.(Suzuki et al. 2012) and its Table 7.

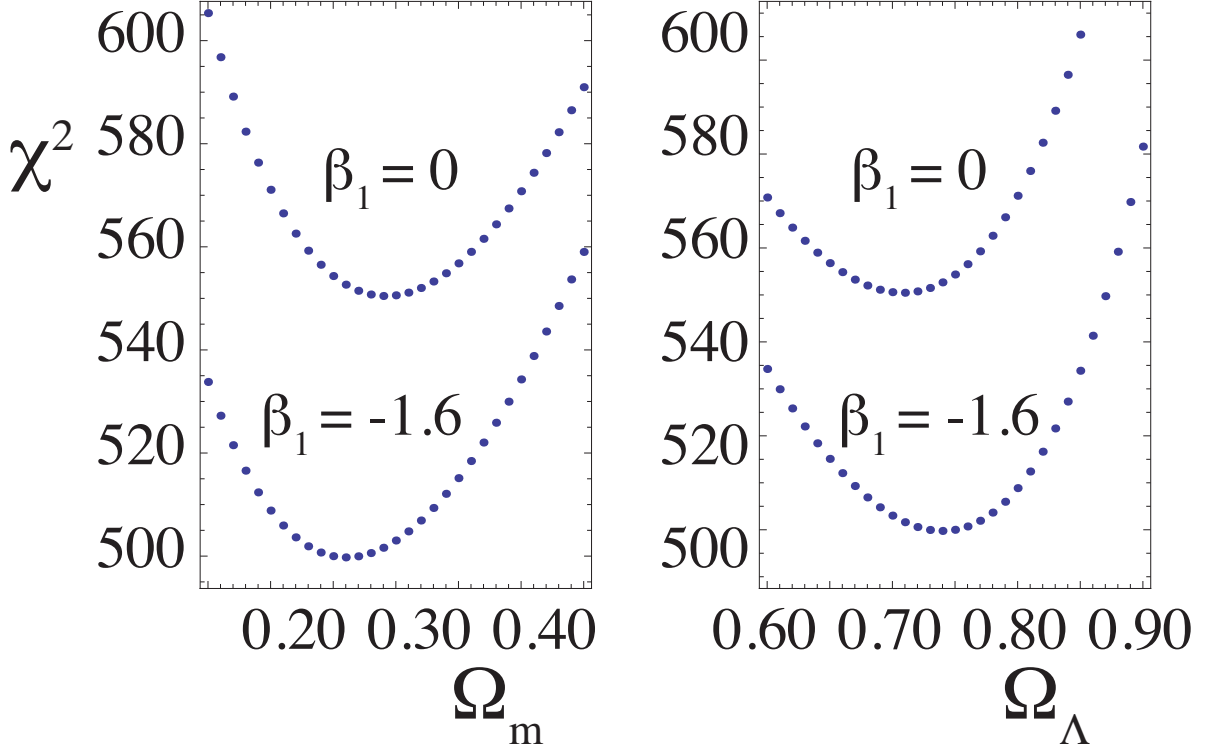


Fig. 3.— Left panel: χ^2 versus Ω_m without *color* \times *redshift* parameter ($\beta_1 = 0$, upper points) and including it ($\beta_1 = -1.6$, lower points). Right panel: Same as left with Ω_Λ on the x -axis. Fits use $\Omega_m + \Omega_\Lambda = 1$ and best-fit values of the remaining parameters point-by-point.

procedures that involve the *FLRW* model and its z dependence, producing hidden redshift dependence in analysis. Once again we refer to the somewhat crude systematic error study presented in Section 3

3. Discussion

The estimated statistical significance of the *color* \times *redshift* correlation is sufficiently high that ordinary confidence levels fail to express it. The correlation is very robust and too large to be attributed to outliers.

We mentioned one test in Section 1.2. For an independent test we computed the difference $\Delta\chi^2(N) = \chi^2(\beta_1 = 0, N) - \chi^2(\beta_1, N)$, where N points were selected on the basis of their uncertainty-weighted residuals relative to the $\beta_1 = 0$ fit of the full set, as follows. The data was sorted in order of decreasing $\delta_{\mu,1}^2/\sigma_1^2 > \delta_{\mu,2}^2/\sigma_2^2 > \dots \delta_{\mu,N}^2/\sigma_N^2$. Rejecting the first N points and comparing $\beta_1 = 0$ with $\beta_1 \neq 0$ produces $\Delta\chi^2(N)$. Thus $\Delta\chi^2(3)$ omits the three largest weighted

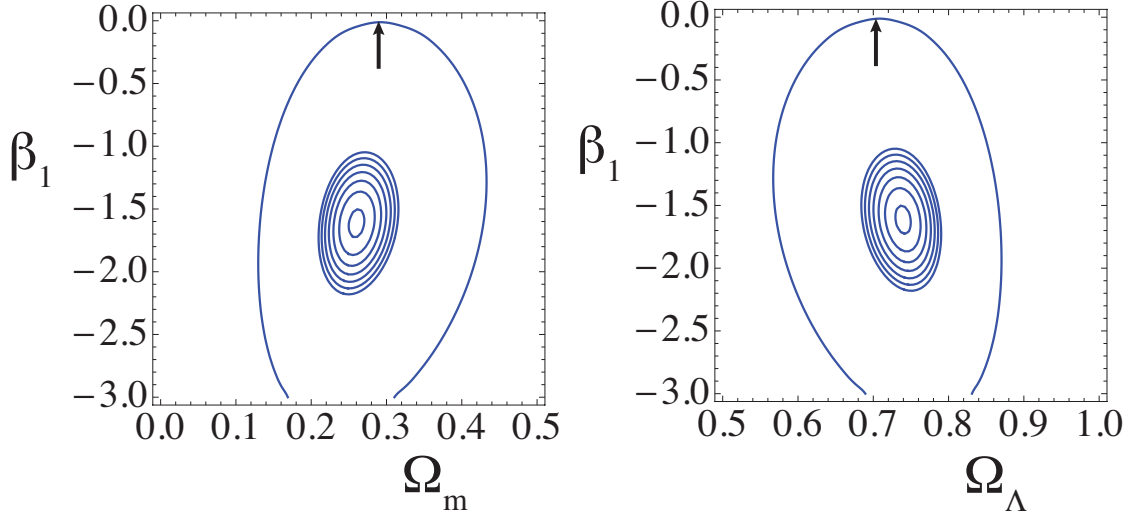


Fig. 4.— *Left Panel:* Contours of constant χ^2 in the (Ω_m, β_1) plane. Contours start at $\chi_{min}^2 + 1 = 500$ and are separated by one unit, except for the outermost contour, corresponding to $\chi^2 = 550$ that intersects $(\beta_1 = 0, \Omega_m = 0.29)$, as indicated by the small arrow. *Right panel:* Same as left using the $(\Omega_\Lambda, \beta_1)$ plane with the outermost intersecting at $(\beta_1 = 0, \Omega_\Lambda = 0.71)$. Fits use $\Omega_m + \Omega_\Lambda = 1$ and best-fit values of the remaining parameters point-by-point.

residuals, and so on. This procedure is statistically unfair, and strongly biased in favor of the $\beta_1 = 0$ hypothesis, because at each N it selects the data to maximally confirm the hypothesis. On the other hand, if outliers were causing the correlation we observe, then we would expect to see $\Delta\chi^2(N)$ suddenly decrease to zero for some N . Instead $\Delta\chi^2(N)$ decreased smoothly with N from $\Delta\chi^2(0) = 51$ to $\Delta\chi^2(50) = 20$. The computation was stopped at $N = 50$, where $\chi^2(\beta_1 = 0, N = 50)/df$ has been artificially decreased by rejecting points from about $550/(550-6)$ to about $300/(500-6)$, a radical reduction.

Two classes of questions naturally arise:

- **Data Selection and Processing:** It would be interesting to know whether the effect might possibly hinge on details of data selection or systematic errors. We have limited information, and our studies are naturally restricted to the compilation as published. Selection effects certainly pose a non-trivial question. (Amanullah et al. 2010) give an extensive discussion of the fact that intrinsically brighter supernovae tend to be selected at large z . Yet if the usual color and stretch corrections would describe the sources, we cannot see how selection on the population would produce a false $c \times z$ correlation or a signal in a likelihood (χ^2) test. More subtle issues would best be pursued by those who control the original data. Regarding data selection on quality basis, powerful consistency checks have already been presented in the

references of (Kowalski et al. 2008; Amanullah et al. 2010; Suzuki et al. 2012). For example, the distributions and correlations of the residuals with $\alpha_0 \beta_0$, δ_0 , M_B , z from both the full compilations as well as subdivided in the Union studies were shown to be within statistical expectations.

The Union 2.1 systematic errors were produced with iterations with the model that was fit. That leads to a question whether our revised model might exploit some anomaly in the systematic errors. Ruling out this possibility explains the relevance of our correlation r_{SN} that included no errors. This is also the motivation for repeating our fits with raw magnitude errors, which at least ought to be unbiased. We considered introducing additional terms into the systematic errors to weaken the *color – redshift* effect, but soon realized it would be trivial and irresponsible to make it go away by design. Instead we explored a simple proxy for systematic errors which padded the systematic magnitude errors with a parameter ξ by the rule $(\Delta m_B)^2 \rightarrow (\Delta_\xi m_B)^2 = (\Delta m_B)^2 + \xi^2$. The procedure tests the possibility that points of small error might dominate the fit and skew the results. By not attempting to adjust errors to fit the *FLRW* model, the padded magnitude uncertainties also tend toward an unbiased procedure. The recomputed best-fits comparing $\beta_1 \neq 0$ and $\beta_1 = 0$ yielded a smooth and nearly monotonic variation of $\Delta\chi^2 = 51$ ($\xi = 0.001$) to $\Delta\chi^2 = 35.5$ ($\xi = 0.2$). The range of ξ spans the differences of $\overline{\Delta m_{B raw}} \sim 0.08$ to $\overline{\Delta \mu_{B systematic}} \sim 0.22$. The test does not support the possibility systematic error assignments might cause the correlation.

- **Physical Interpretation:** Assuming the *color – redshift* effect has an astrophysical origin, we use the following facts to interpret it. When introducing his color correction parameter Tripp (1998) wrote that “To accommodate different amounts of reddening observed in the 29 supernovae of Hamuy et al. (1996), *arising either from an intrinsically reddened supernova or from intervening dust in the parent galaxy*, we introduce... another phenomenological parameter R .” The R parameter is essentially β_0 . Continuing, Tripp wrote that “ by applying the same type of color correction to cosmological supernovae *even without knowing whether reddening is intrinsic or due to dust*, one will be able to completely standardize the light output of each explosion and thereby get substantially better values for q_0 and the mass density Ω_m of the universe”. (Italics are ours.) Tripp’s presentation cited van den Bergh (1995), who noticed that model calculations showed a bluer-brighter correlation. van den Bergh suggested an effective magnitude parameter which to a first approximation would be independent of both reddening and the supernova model. Even earlier Branch and Tammann (1992) had noticed a puzzle that R found in data was much smaller than expected from models and general considerations involving dust. Besides the work of (Tripp 1998; Tripp & Branch 1999) the need for *color* corrections in modern *SNe* analysis was noted by (Riess Press & Kirshner 1996; Guy et al. 2007) and appears in all Union compilations.

In our generalization of $\beta \rightarrow \beta_0 + \beta_1 z$, a positive value of β_1 would increase the effect with increasing z , as expected from accumulating effects of dust. Instead we find $\beta_1 < 0$, which

we cannot understand would be caused by increasing amounts of dust with distance. The *color – redshift* effect is, however, consistent with evolution of the sources. This may be related to the long-standing puzzle highlighted by Branch and Tammann (1992) and others that β from data fits is smaller than expected. If the *color – redshift* effect is not taken into account, its trend will generally decrease the value found in a one constant (β or R) model that is fit over a range of redshifts. As consistent, our $z \sim 0$ parameter $\beta_0 = 2.62$ is somewhat larger than the value found setting $\beta_1 = 0$. The general sense of the correction from β_1 is this: modifying the known rule that bluer sources are brighter, the proportionality tends to decrease with larger z . Extrapolating naively to large enough z , the bluer-brighter relation would eventually reverse. Since reversal is implausible, it appears the bluer-brighter relation probably saturates at large z , close to the largest redshifts observed so far. Whether this is a fact of supernovae or how the supernovae are observed is unknown.

We conclude that the *color – redshift* effect is evidence for significant evolution of Type Ia supernovae or modifications of their environments with increasing redshift. Other explanations may exist. It seems premature to attempt the last word on the highly significant trend we have found.

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